

AVERAGED EQUATIONS OF RADIATION TRANSFER AND THEIR UTILIZATION IN THE SOLUTION OF GASDYNAMIC PROBLEMS

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Equations of radiative transfer were integrated (averaged) over angles and energies of quanta using the true directional profile and the true emission spectrum in each point of space. A multigroup system of averaged equations of the "back and forth" type was obtained (also including the equations of the spherical case) which was equivalent to the initial system of equations of transfer, i. e. this system leads to the same values of average group or integral intensity of radiation (for the instant of averaging). A method is proposed for utilization of this system simultaneously with equations of gasdynamics (dimensionless coefficients in averaged equations are retained, for example, for a given value of optical thickness in the intervals between averaging, then a recalculation is made). Even in the case of relatively frequent averaging, the laboriousness of solving nonsteady-state gasdynamic problems (and steady-state problems which are solved by the iteration or relaxation method) decreases sharply. The comparison of the dimensionless coefficients in the averaged equations provides an objective "automated" evaluation of the necessary frequency for carrying out the averaging.

It is pointed out that this method can be generalized to the non-one-dimensional case, and also to the case where local thermodynamic equilibrium is not present.

Particular attention is devoted to the problem of averaging the transfer of radiation from an external source which heats and vaporizes material. The effectiveness of the averaging method can be quite high because in such problems the heating processes develop (from gasdynamic point of view) quite slowly. Acoustic disturbances have time for multiple propagation through the relatively thin layer of the heated material. This results in slowly varying profiles of temperature and density. The spectrum and the directional profile of the source can differ substantially from the spectrum and the directional profile of the characteristic emission of the material. Therefore, the averaging of the radiative transfer from the source is carried out separately from the averaging of the characteristic emission (generated in the heated medium). In many cases the characteristic emission is negligibly small in general. Then further simplifications are possible and a strong analogy exists between the problems of action on material of radiation from a continuous spectrum and radiation which is monochromatic.

1. For the case where the parameters of the material are constant on certain planes or spherical surfaces with a common center and where the scattering and the time of propagation of radiation through the heated or cooled volume are neglected, equations of radiative transfer have the following form [1]:

$$\mu \frac{\partial J_\epsilon}{\partial r} + \frac{(\nu - 1)(1 - \mu^2) \partial J_\epsilon}{2r \partial \mu} = -k_\epsilon (J_\epsilon - B_\epsilon) \quad (1.1)$$

Here J_ϵ is the intensity of radiation increased π times and referred to the unit interval of energy of quanta (and not of frequencies, as it is usually done), r is Euler's coordinate, $\nu = 1$ and 3 in the plane and spherical cases, k_ϵ is the linear spectral absorp-

tion coefficient corrected for the forced emission, $\mu = \cos \theta$, where θ is the angle of intersection of the ray with the radius or the perpendicular to the planes with equal values of parameters, B_ε is the Planck function

$$B_\varepsilon = \frac{15}{\pi^4} \frac{\sigma \varepsilon^3}{\exp(\varepsilon/T) - 1}, \quad \int_0^\infty B_\varepsilon d\varepsilon = \sigma T^4 = B \quad (1.2)$$

(σ is the Stefan-Boltzmann constant, T is the temperature).

On the boundary surface (plane or sphere) the radiation intensity from some external source (designated by the index "zero") is given for all rays entering into the volume, i. e. for $0 \leq \mu \leq 1$ or $-1 \leq \mu \leq 0$. For each value of μ the quantity J_ε is the same on the entire boundary surface. Then the radiation is characterized by a symmetrical directional profile $J_\varepsilon = J_\varepsilon(\varepsilon, \mu, r, t)$ and we can utilize (1.1).

In the plane case it is easy to write (1.1) in the integral form [1 - 4]

$$J_\varepsilon = J_\varepsilon^\circ \exp\left(-\frac{\tau_\varepsilon}{\mu}\right) + \int_0^{\tau_\varepsilon} B_\varepsilon(\tau_\varepsilon') \exp\left(-\frac{\tau_\varepsilon - \tau_\varepsilon'}{\mu}\right) d\tau_\varepsilon' \quad (1.3)$$

The spectral optical thickness τ_ε is determined by the following expression:

$$\tau_\varepsilon = \int_{r_0}^r k_\varepsilon dr = \int_0^m \kappa_\varepsilon r^{-(\nu-1)} dm \quad \left(\kappa_\varepsilon = \frac{k_\varepsilon}{\rho}\right)$$

Here κ_ε is the spectral mass absorption coefficient, ρ is the density and m is the Lagrange mass coordinate.

Equation (1.1) is linear with respect to J_ε . Therefore, in each point, J_ε represents a superposition of a quantity proportional to the intensity of radiation of the source J_ε° (taking into account its reduction due to absorption from the boundary to the given point) and a quantity which is determined by the characteristic emission generated in the medium itself. This is clearly evident from (1.3).

Having solved (1.1), or in the plane case (1.3), we find J_ε in each point r for all μ and we calculate the average characteristics of the radiation field: spectral unilateral flux density of radiation

$$q_\varepsilon^+ = 2 \int_0^1 J_\varepsilon \mu d\mu, \quad q_\varepsilon^- = 2 \int_{-1}^0 J_\varepsilon \mu d\mu \quad (1.4)$$

average (with respect to angles) spectral unilateral intensity of radiation, increased π times

$$U_\varepsilon^+ = \int_0^1 J_\varepsilon d\mu, \quad U_\varepsilon^- = \int_{-1}^0 J_\varepsilon d\mu \quad (1.5)$$

group unilateral flux density of radiation

$$q_i^\pm = \int_{\varepsilon_1}^{\varepsilon_2} q_\varepsilon^\pm d\varepsilon \quad (1.6)$$

group average unilateral intensity of radiation

$$U_i^\pm = \int_{\varepsilon_1}^{\varepsilon_2} U_\varepsilon^\pm d\varepsilon \quad (1.7)$$

Here ε_1 and ε_2 are the upper and lower limits of the i th group of quanta (for simplicity of notation the index i has been left out in the case of ε_1 and ε_2 in (1.6) and (1.7), and will also be omitted in the following text). If the group consists of several subgroups

which are not in contact with each other, then the modification of (1.6) and (1.7), and also the equations presented below where integration with respect to ε is encountered, are obvious.

If $\varepsilon_1 = 0$ and $\varepsilon_2 = \infty$, i. e. integration is carried out over the entire spectrum, then q^\pm and U^\pm (without the index i) will be called integral unilateral flux densities and average intensities.

The total (in both directions) spectral and integral flux densities and average intensities are determined in the following manner: (1.8)

$$q_\varepsilon = q_\varepsilon^+ + q_\varepsilon^-, \quad q = \sum_i (q_i^+ + q_i^-), \quad U_\varepsilon = U_\varepsilon^+ + U_\varepsilon^-, \quad U = \sum_i (U_i^+ + U_i^-)$$

We introduce a function characterizing the directional profile of the radiation

$$\psi_\varepsilon^\pm = J_\varepsilon / U_\varepsilon^\pm \tag{1.9}$$

and a function characterizing its spectral composition in the i th group

$$(\varphi_\varepsilon^\pm)_i = U_\varepsilon^\pm / U_i^\pm \tag{1.10}$$

By virtue of (1.5) and (1.7) we have

$$\int_0^1 \psi_\varepsilon^+ d\mu = 1, \quad \int_{-1}^0 \psi_\varepsilon^- d\mu = 1, \quad \int_{\varepsilon_1}^{\varepsilon_2} (\varphi_\varepsilon^\pm)_i d\varepsilon = 1 \tag{1.11}$$

From (1.4), (1.5), (1.9) and (1.10) we obtain

$$q_\varepsilon^\pm = 2c_\varepsilon^\pm U_\varepsilon^\pm \tag{1.12}$$

Here c_ε is the unilateral spectral average (with respect to angle) cosine

$$c_\varepsilon^+ = \int_0^1 \psi_\varepsilon^+ \mu d\mu, \quad c_\varepsilon^- = \int_{-1}^0 \psi_\varepsilon^- \bar{\mu} d\mu \tag{1.13}$$

Integrating (1.12) with respect to ε , we obtain

$$q_i^\pm = 2c_i^\pm U_i^\pm \tag{1.14}$$

here c_i^\pm are group unilateral average cosines

$$c_i^\pm = \int_{\varepsilon_1}^{\varepsilon_2} c_\varepsilon^\pm (\varphi_\varepsilon^\pm)_i d\varepsilon \tag{1.15}$$

2. The equation which describes the energy change with respect to time for a unit mass due to expansion or contraction of the gas, and also due to energy supply as a result of absorption or due to energy losses as a result of emission of radiation, has the form

$$\frac{\partial e}{\partial t} + p \frac{\partial v}{\partial t} = \dot{j} = \sum_i \dot{f}_i, \quad \dot{f}_i = \int_{\varepsilon_1}^{\varepsilon_2} \dot{f}_\varepsilon d\varepsilon \quad \left(v = \frac{1}{\rho} \right) \tag{2.1}$$

Here p is the pressure, v is the specific volume, ρ is the density, \dot{f}_ε , \dot{f}_i and \dot{f} are the spectral, group and integral intensities of generation ($\dot{f} > 0$) or losses ($\dot{f} < 0$) of energy by a unit mass. From the equation of transfer we can obtain directly

$$\dot{f}_\varepsilon = 2\kappa_\varepsilon (U_\varepsilon^+ + U_\varepsilon^- - 2B_\varepsilon), \quad \dot{f}_i = 2(\langle \kappa \rangle_i^+ U_i^+ + \langle \kappa \rangle_i^- U_i^- - 2\kappa_i^p B_i) \tag{2.2}$$

Here κ_i^p is Planck's group coefficient of absorption

$$\kappa_i^p = \int_{\varepsilon_1}^{\varepsilon_2} \kappa_\varepsilon \left(\frac{B_\varepsilon}{B_i} \right) d\varepsilon = \int_{\varepsilon_1}^{\varepsilon_2} \kappa_\varepsilon (\varphi_\varepsilon^p)_i d\varepsilon \tag{2.3}$$

For $\varepsilon_1 = 0$ and $\varepsilon_2 = \infty$ it transforms into the usual (in our terminology integral) Planck coefficient. B_i is the group flux density of the radiation for the black body, $\langle \kappa \rangle_i^\pm$ is the unilateral group coefficient of absorption

$$B_i = \int_{\varepsilon_1}^{\varepsilon_2} B_\varepsilon d\varepsilon, \quad \langle \kappa \rangle_i^\pm = \int_{\varepsilon_1}^{\varepsilon_2} \kappa_\varepsilon (\varphi_\varepsilon^\pm)_i d\varepsilon \quad (2.4)$$

If the spectrum φ_ε^\pm coincides with Planck's spectrum (in the case of local temperature T), then $\langle \kappa \rangle_i^\pm = \kappa_i^p$. If this is not the case, we talk about the distortion of the spectrum with respect to Planck's spectrum and introduce a dimensionless coefficient which characterizes such distortion

$$\eta_i^\pm = \langle \kappa \rangle_i^\pm / \kappa_i^p \quad (2.5)$$

From this

$$f_i = 2\kappa_i^p (\eta_i^+ U_i^+ + \eta_i^- U_i^- - 2B_i) \quad (2.6)$$

It follows from (2.6) that only integral characteristics of the radiation field are required for gasdynamic calculations. We shall attempt to obtain equations which describe the change of just these quantities with respect to mass.

3. The equation (1.1) is somewhat transformed

$$\frac{\partial (\mu J_\varepsilon r^{\nu-1})}{r^{\nu-1} \partial r} + \frac{(\nu-1)}{2r} \frac{\partial (J_\varepsilon (1-\mu^2))}{\partial \mu} = -k_\varepsilon (J_\varepsilon - B_\varepsilon) \quad (3.1)$$

The equation (3.1) is integrated from $\mu = -1$ to $\mu = 0$ and from $\mu = 0$ to $\mu = 1$. As a result we obtain

$$\frac{\partial (c_\varepsilon^\pm U_\varepsilon^\pm r^{\nu-1})}{r^{\nu-1} \partial r} + \frac{(\nu-1) g_\varepsilon^\pm U_\varepsilon^\pm}{2r} = -k_\varepsilon (U_\varepsilon^\pm - B_\varepsilon) \quad (3.2)$$

In the derivation of (3.2) it was assumed that for $\mu \rightarrow 0$ the quantity J_ε is finite and that g_ε^\pm , the spectral unilateral coefficient of "sphericity", is determined in the following manner: $U_\varepsilon^\pm g_\varepsilon^\pm = \mp J_\varepsilon$ or $g_\varepsilon^\pm = \mp \psi_\varepsilon^\pm$ (for $\mu = 0$)

$$(3.3)$$

It characterizes the "efflux" and "influx" of radiation from one group (with the index "plus") to another (with the index "minus") and back.

The reason for the appearance of this coefficient is that in the spherical (and cylindrical) case for a fixed ray the angle θ of its intersection with the radius changes all the time. This is actually reflected in (1.1). If the ray is directed toward the center (index minus), then the angle of intersection gradually decreases from $\theta = \pi$ to $\theta = \pi/2$ (μ increases from $\mu = -1$ to $\mu = 0$), after this the ray falls into a group with the index plus ("it turns back").

A system of equations close to (3.2) was obtained by the author of [5] (he called it "a quasi-diffusion approximation") by integration of the kinetic equation with respect to μ from $\mu = -1$ to $\mu = 1$. The second equation in [5] was obtained by means of the same integration but with preliminary multiplication by μ . These equations contain as independent variables the total flux density of radiation q_ε and the total average intensity U_ε , and not the corresponding unilateral quantities as in our case (in [5] the averaging of the kinetic equation was carried out with application to problems of neutron propagation. In this case scattering was taken into account. However, it is natural that the same considerations can be utilized for problems of propagation of radiation).

Equations (3.2) differ from the ones found in [5], and are closer to the system of equations of the "back and forth" type obtained earlier in the plane case by Schwarzschild

and Mustel' and in the approximation of straight rays in [1 - 4]. In these approximations the form of the directional profile is prescribed in advance and does not change in the entire domain (which, of course, strictly speaking, is impossible with the exception of the black body case with complete equilibrium of material and radiation, where $q_e^+ = -q_e^- = B_e$ and $q_e = 0$). Therefore, constant values are obtained for spectral and integral average cosines c_e^\pm and c^\pm , namely $1/2$, $2/3$ and 1 (respectively). The system (3.2) obtained by us and also the quasi-diffusion approximation are exact in the sense that for true values of the average cosine and coefficients of sphericity which are found from the true directional profile, we obtain the same quantity U_e^\pm as is obtained directly from the equation of transfer (1.1).

It appears that system (3.2) is more convenient than the system of quasi-diffusion approximation because each of the equations contains only one of the sought functions U_e^+ or U_e^- which is important because the boundary conditions for them are given on different boundaries. Analogous averaging over angles within the limits of the half-space was carried out in the recently published papers [15, 16].

We obtain a multigroup system of equations. We integrate (3.2) within the limits of each group

$$\frac{\partial (c_i^\pm U_i^\pm r^{\nu-1})}{r^{\nu-1} \partial r} + \frac{(\nu-1) g_i^\pm U_i^\pm}{r} = - \langle k \rangle_i^\pm U_i^\pm - k_i^p B_i \quad (3.4)$$

Here $\langle k \rangle_i^\pm$ is the linear average absorption coefficient, k_i^p is Planck's linear absorption coefficient, related to $\langle \kappa \rangle_i^\pm$ and to κ_i^p by the simple relationships

$$\langle k \rangle_i^\pm = \langle \kappa \rangle_i^\pm \rho, \quad k_i^p = \kappa_i^p \rho \quad (3.5)$$

The group unilateral coefficient of sphericity is determined in the following manner:

$$g_i^\pm = \int_{\epsilon_1}^{\epsilon_2} g_\epsilon^\pm(\varphi_\epsilon^\pm)_i d\epsilon \quad (3.6)$$

Using the distortion coefficients of Planck's spectrum determined according to (2.8), and changing over to Lagrange mass coordinates, we obtain from (3.4)

$$\frac{\partial (c_i^\pm U_i^\pm r^{\nu-1})}{\partial m} + \frac{(\nu-1) g_i^\pm U_i^\pm}{\rho r} = - \kappa_i^p (U_i^\pm \eta_i^\pm - B_i) \quad (3.7)$$

The obtained system of equations is exact in the sense that if in each point the true value of c_i^\pm , g_i^\pm and η_i^\pm is utilized, we obtain from (3.7) the same values U_i^\pm as from the initial equation of transfer (1.1).

It is frequently the case that the characteristic emission in the volume under examination is negligibly small in comparison to the radiation of the external source. In this connection the spectrum and the directional profile of radiation are determined by the spectrum and the directional profile of the source, at least near the boundary of the heated volume. The partition of radiation into groups according to energies of the quanta and the separate integration over the angles of radiation directed along the negative and positive r -axis is due to the desire to take into account as accurately as possible the spectral composition of radiation and its anisotropy. From the same point of view it is natural to bring up separate integration for the radiation of the source and the characteristic emission.

The subscript zero will be used to designate quantities related to characteristic emission. The quantity J_{e0} will be determined from (1.1) for $(J_e)^\circ = 0$. The quantity J_e will be determined from the same equation, but for a value of $B_e = 0$. In exactly

the same manner in the averaged equations the quantity $U_{\epsilon 0}^{\pm}$ will be determined from (3.2) for $(U_{\epsilon 0}^{\pm})^{\circ} = 0$ and the quantity $U_{\epsilon \pm}$ will be determined from the same equation, but for a value of $B_{\epsilon} = 0$. Finally, the quantities $U_{i 0}^{\pm}$ will be found from (3.4) for $(U_{i 0}^{\pm})^{\circ} = 0$, and the quantities U_i^{\pm} with $B_i = 0$. Relationships presented above which determine $c_{\epsilon \pm}$, c_i^{\pm} etc., can also be used for quantities designated by the index zero; we present only the relationship which replaces (2.4)

$$f_i^{\circ} = 2(\langle \kappa_0 \rangle_i^+ U_{i 0}^+ + \langle \kappa \rangle_i^+ U_i^+ + \langle \kappa_0 \rangle_i^- U_{i 0}^- + \langle \kappa \rangle_i^- U_i^- - 2\kappa_i^p B_i) \quad (3.8)$$

Averaged equations of transfer can be utilized for various purposes. The compact notation greatly facilitates the written formulation of the gasdynamic problem and its qualitative analysis. It is noted that certain general properties of a gasdynamic system of equations and a system of equations of transfer (formulation of boundary conditions, determination of initial values, etc.) have already been examined by the author of [6] with the aid of multigroup averaged equations in the "back and forth" type. In this connection the method itself for obtaining the coefficients in the averaged equations was not refined and the work was limited to the plane case only. We note that practically the entire analysis carried out in [6], is also valid for the equations obtained here which were averaged in an exact manner (this includes also the spherical case).

The averaged equations make it easier to search for self-similar solutions, etc. However, the main thing is that by means of these equations it is possible to create an effective method of calculation for gasdynamic problems in which the energy transfer is realized through radiation of the continuous spectrum (*).

At some moment of time t_1 let the distribution of temperature $T(m)$ and of density $\rho(m)$ be known. After finding J_{ϵ} from (1.1), we calculate U_{ϵ}^{\pm} from (1.5) and determine ψ_{ϵ}^{\pm} from (1.9). From (1.13) we find c_{ϵ}^{\pm} , from (3.3) and (1.7) we determine g_{ϵ}^{\pm} and U_i^{\pm} . Further we determine $(\varphi_{\epsilon}^{\pm})_i$ from (1.10), c_i^{\pm} from (1.15), g_i^{\pm} from (3.6), $\langle \kappa \rangle_i^{\pm}$ from (2.4) and η_i^{\pm} from (2.5). Utilizing now the dimensionless coefficients g_i^{\pm} , c_i^{\pm} and η_i^{\pm} we shall solve the averaged equations of transfer (3.7) simultaneously with the gasdynamic equations with these coefficients in the next moments of time. In this connection, naturally, the quantities κ_i^p and B_i each time corresponds to new distributions of T and ρ . At some later moment of time (after a certain number of calculated layers) t_2 the entire averaging procedure is carried out again and we repeat from t_1 and t_2 the whole gasdynamic calculation with average coefficients obtained by interpolation between values calculated at moments t_1 and t_2 (recalculation).

However, the question arises about the method of retaining of dimensionless coefficients in the intervals between the moments of averaging. The point is that these coefficients are a function of two variables, while at the moment of averaging they can be found only as a function of one variable m . Only after repeated averaging it is possible to determine their derivative with respect to time. Furthermore, it is not obvious in advance that the averaging will be effective (it is possible to perform it quite infrequently) if we select as variables m and t themselves. By means of careful analysis of a concrete problem and also as a result of accumulated experience in performing the averaging for the given type of problems it is possible to find such combinations of independent

*) During the discussion of this paper at the Institute of Applied Mathematics of the Academy of Sciences, USSR, it became known that closely similar ideas were developed by authors of [5, 15, 16].

variables for which the dimensionless coefficients will be primarily functions of only one variable and will have only weak dependence on the second variable. As one of such variables we can point out Planck's group optical thickness which is determined by the following relationship:

$$\tau_i^p = \int_0^m \kappa_i^p r^{-(\nu-1)} dm \tag{3.9}$$

It is natural that such a selection of the "major" variable is not unique. We can propose other methods. This problem will be analyzed using an example where the characteristic emission of the medium can be neglected completely and where it is possible to limit oneself to the examination of equations of radiative transfer from the source.

4. Let us examine the plane case. For simplicity it will be assumed that the source acts only from one side and the indices "plus" and "minus" are omitted. From (1.3) and (1.5) we obtain

$$U_\epsilon = U_\epsilon^\circ \int_0^1 \psi_\epsilon^\circ \exp\left(-\frac{\tau_\epsilon}{\mu}\right) d\mu \tag{4.1}$$

Here ψ_ϵ° is the initial directional profile. Let the source be isotropic: $\psi_\epsilon^\circ = 1$.

In Fig.1 the directional profile of radiation is shown for various optical thicknesses τ_ϵ from the boundary of the material on which the radiation falls. It is evident that a greater and greater role is played by straight rays ($\mu \approx 1$). The spectral values of the average intensity of radiation and of the average cosine vary with τ_ϵ in the following manner:

$$U_\epsilon = U_\epsilon^\circ E_2(\tau_\epsilon), \quad q_\epsilon = q_\epsilon^\circ 2E_3(\tau_\epsilon), \quad c_\epsilon = E_3(\tau_\epsilon)/E_2(\tau_\epsilon) \tag{4.2}$$

Here $E_n(x)$ is an integral exponential function of the n th order [1]. Starting from the value of this function at $x = 0$ and the asymptotic expression at large x , namely

$$E_n(0) = (n - 1)^{-1}, \quad E_n(x) = x^{-1} \exp(-x) \tag{4.3}$$

we can construct a simple approximate analytical expression for $E_n(x)$ for the entire range $0 \leq x \leq \infty$

$$E_n(x) = (n - 1 + x)^{-1} \exp(-x) \tag{4.4}$$

A comparison of (4.4) with exact values shows that the difference does not exceed 20% for $n = 2$ and decreases with increasing n and also with $x \rightarrow 0$ and $x \rightarrow \infty$.

It is convenient to represent tables of $E_n(x)$ in the form of relatively small corrections to (4.4). From (4.2) and (4.4) we obtain that the average cosine varies with the spectral optical thickness in approximately the following manner:

$$c_\epsilon = (\tau_\epsilon + 1) / (\tau_\epsilon + 2) \tag{4.5}$$

It is evident that the average cosine c_ϵ will be smooth and a relatively weakly varying function of the spectral optical density τ_ϵ . This is also true for other initial directional profiles of the source ψ_ϵ° , even though in this case it is not possible to obtain

equally simple expressions for $q_\epsilon(\tau_\epsilon)$ as in (4.2). If the directional profile ψ_ϵ° of the source is invariant with time, then the relation $c_\epsilon(\tau_\epsilon)$ is also invariable.

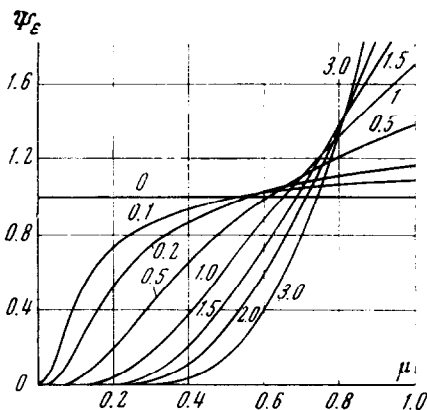


Fig. 1

Having computed the radiation field at some instant, we can determine this relationship and subsequently find q_ε from Eq. (3.4) averaged over angles. This equation in the present case (taking into account only the transfer of radiation from the source, $\nu = 1$) takes the form

$$c_\varepsilon \frac{\partial q_\varepsilon}{\partial \tau_\varepsilon} = -q_\varepsilon, \quad q_\varepsilon = q_\varepsilon^\circ \exp\left(-\int_0^{\tau_\varepsilon} \frac{d\tau_\varepsilon}{c_\varepsilon}\right) \quad (4.6)$$

Here the relationship (1.12) was taken into account.

A comparison of this equation with the first term of (1.3) shows that the problem of action of radiation from an undirected source is reduced to the problem of action from a directed source, however, in each point the angle of intersection of the ray and the x -axis is varied in a predetermined manner.

As far as the spherical case is concerned, the directional profile in each point is determined not only by the initial profile ψ_ε° and the quantity τ_ε , but also by the radius r itself. However, in certain problems, for example action of radiation on the surface of a sphere, its evaporation, and motion of vapors, heated by radiation, away from the evaporating sphere, a quasi-steady-state mode can arise [7], where all parameters, and consequently also τ_ε , depend only on the radius r . Thus, the possibility of generalizing the problem [7] to the case of the continuous spectrum is also obvious. The closer the motion is to quasi-steady-state, the less frequently will it be necessary to perform the averaging in the non-steady-state problem (this method can be utilized also in steady-state problems which are solved by the iteration or relaxation method).

The equation of transfer (4.6) averaged over the energies of the quanta, assumes the form

$$c_i^\pm \frac{\partial q_i^\pm}{\partial m} = -\langle \kappa \rangle_i^\pm q_i^\pm \quad (4.7)$$

(It represents a particular case of (3.7).) On the surface of the irradiated substance the radiation spectrum is known. It coincides with the spectrum of the source $(\varphi_\varepsilon^\circ)_i$. Therefore, for all temperatures and densities it is possible to determine the surface coefficient of absorption

$$\langle \kappa \rangle_i^\circ = \int_{\varepsilon_1}^{\varepsilon_2} \kappa_\varepsilon (\varphi_\varepsilon^\circ)_i d\varepsilon \quad (4.8)$$

Having solved the transfer equation and knowing the spectrum in all points, it is possible to determine a coefficient which takes into account the distortion of such a spectrum in comparison to the initial spectrum

$$\xi_i^\circ = \langle \kappa \rangle_i / \langle \kappa \rangle_i^\circ \quad (4.9)$$

The indices plus or minus are omitted because we are examining a unilateral source. Utilizing (4.9), we write instead of (4.7)

$$c_i \frac{\partial q_i}{\partial \tau_i^\circ} = -\xi_i^\circ q_i, \quad \tau_i^\circ = \int_0^m \langle \kappa \rangle_i^\circ dm \quad (4.10)$$

Sometimes it is inconvenient to use the surface average because it takes into account the absorption of the kind of quanta which have small penetrating capability into the material, carry a small amount of energy and are therefore not characteristic for the problem as a whole. Then it is possible to utilize a "supporting" coefficient of absorption $\kappa^\circ = \kappa(\langle \varepsilon \rangle^\circ)$ for the average energy of the quanta of the source

$$\langle \varepsilon \rangle^\circ = \int_{\varepsilon_1}^{\varepsilon_2} \varepsilon \varphi_\varepsilon^\circ d\varepsilon \quad (4.11)$$

In a corresponding manner it is easy to change the determination of ξ_i° and τ_i° .

In many cases the spectrum of the source is constant (in spite of changes in flux density). This applies to a source of Planck's spectrum which is at constant temperature but has finite dimensions and approaches the irradiated surface (an exploding laboratory source [8, 9] – a shock wave which propagates through a tube filled with the working gas; due to the change of the geometric factor the flow density increases continuously). This applies also to a source radiating in the Rayleigh-Jeans region at constant temperature. The spectrum of this source has the form

$$\varphi_\varepsilon^\circ = 4(\varepsilon/\varepsilon_m)^3 \varepsilon_m^{-1} \quad (4.12)$$

Here ε_m is the maximum energy of quanta from the source. Other situations are also possible where the spectrum $\varphi_\varepsilon^\circ$ is not changed.

Even in this case it is not possible to make up in advance tables of distortion coefficients ξ_i° and of group average cosines c_i as functions of group optical thickness τ_i° . The point is that for each energy of quanta ε , the value of the spectral coefficient of absorption κ_ε in the general case depends on temperature T and density ρ in its own way and not in the same way as κ_ε depends on T and ρ for another energy of quanta, and also not in the same way as $\langle \kappa \rangle_i^\circ$ or $\kappa(\langle \varepsilon \rangle_i^\circ)$.

5. Within the limits of the given group let κ_ε depend on T and ρ in the same (even though completely arbitrary) manner

$$\kappa_\varepsilon = K_i(\varepsilon) G_i(T, \rho) \quad (5.1)$$

Then we have

$$\langle \kappa \rangle_i^\circ = \langle K \rangle_i^\circ G_i, \quad \langle K \rangle_i^\circ = \int_{\varepsilon_1}^{\varepsilon_2} K_i(\varepsilon) (\varphi_\varepsilon^\circ)_i d\varepsilon \quad (5.2)$$

or

$$\kappa_i^\circ = \kappa(\langle \varepsilon \rangle_i^\circ) = K(\langle \varepsilon \rangle_i^\circ) G_i = K_i^\circ G_i \quad (5.3)$$

In the case of condition (5.1) the group optical thickness is proportional to the spectral optical thickness for any distribution of temperature and density

$$\tau_i = K_i^\circ \int_0^m G_i(T, \rho) dm, \quad \tau_\varepsilon = \frac{K_i(\varepsilon)}{K_i^\circ} \quad (5.4)$$

It is apparent that in this case in some point the radiation spectrum, which is distorted with respect to the spectrum of the source because of absorption in layers of material lying "above" this point, depends only on τ_i and ε . The coefficients in averaged equations obtained by means of integration with respect to ε depend only on τ_i (for a given spectrum of the source). Tables of the dependence of ξ_i and c_i on τ_i can be composed in advance.

We call attention to the fact that here it is not required for functions K_i and G_i themselves to have a special form of any kind. It is also not required for these functions to be the same in different groups. Thus, for one group, with increasing T the transparency may increase, while for another it decreases. However, this must occur for the entire group as a whole. We note that fulfillment of condition (5.1) indicates the same form of $\kappa(\varepsilon)$ inside the given group for all temperatures and densities. This requirement is satisfied for example by a power and piecewise power form of $\kappa(\varepsilon)$ for constant powers and for invariable jumps in coefficients of absorption on the boundaries of subgroups. In jumps on the boundaries of the groups the absorption coefficients in this connection

vary in an arbitrary manner.

The situation which is close to the described limiting case can arise in reality. For the case of completely ionized gas [4] in the short wave part ($\epsilon \gg T$) of the spectrum $\kappa \sim \epsilon^{-3}$, while in the long wave part ($\epsilon \ll T$) we have $\kappa \sim \epsilon^{-2}$. The same is true also

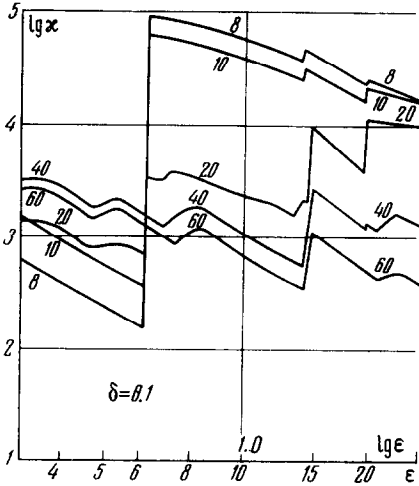


Fig. 2

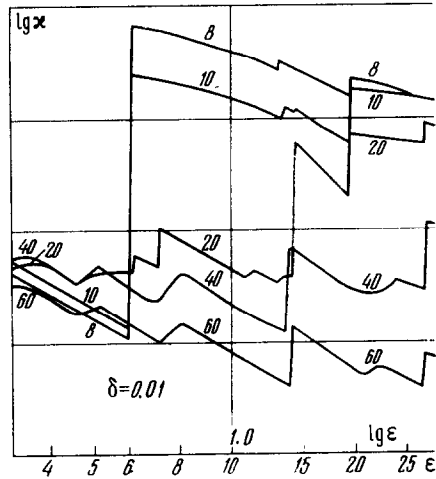


Fig. 3

for continuous absorption of "soft" radiation in gas with single and multiple ionization. This is also true for quite hard radiation which is absorbed by the material due to the

photo effect from deep shells of the atom which are not involved in ionization. In this case the absorption coefficient does not depend at all on T and ρ over a quite wide range of these parameters. Therefore it is possible to formulate only one group with arbitrary variation of $\kappa(\epsilon)$.

In the general case with complex dependence of $\kappa(\epsilon, T, \rho)$ it would appear that a large number of groups is necessary.

However, first of all, adjoining intervals of the spectrum (subgroups) must not necessarily enter into the given group.

Secondly, the requirement for the law of variation of κ with ϵ or T and ρ to be the same, must not necessarily be satisfied for the entire range of these parameters, but only for the most essential region (for example, for the most important parts of the spectrum which are deter-

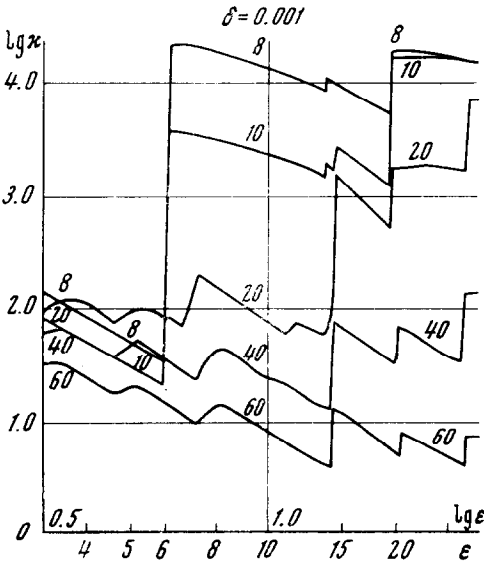


Fig. 4

minally, for the most important parts of the spectrum which are deter-

mined from energetic or some other additional considerations).

Thirdly, the groups themselves can be formulated in such a manner that the relationship (5.1) is satisfied as well as possible.

In Figs. 2 and 4 the behavior of continuous absorption coefficients of aluminum vapors is shown for several temperatures (the numbers correspond to T in $10^{3^{\circ}\text{K}}$) and densities (δ is the relative density; $\delta = \rho / \rho_L$; for $\delta = 1$ the number of atoms and ions is equal to the Loschmidt number, for Al $\rho_L = 1.20 \cdot 10^{-3} \text{g/cm}^3$). At the request of the author the calculation was made by V. A. Onishchuk using the method of Burgess and Seaton [10] (the ionized composition was calculated according to Saha's system of equations [4] by the author and L. P. Markelova).

Here only values of κ are presented in the range of ε to 25 eV, which corresponds to the energy limit of quanta in an explosive source [8, 9] using helium as the working gas (helium has the highest ionization potential (24.6 eV) of all noble gases). Theoretical estimates [4] and experiments [9] show that the maximum temperature for which the screening effect of the shock front still arises is approximately 8 to 9 eV (in this connection the radiation flux density coming from the shock front reaches nearly 150mwatt/cm^2). Consequently, by means of such a source it is possible to heat the vapors up to $80,000^{\circ}\text{K}$.

It is evident from Figs. 2 to 4 that in the examined range of T and ε the number of groups is relatively small (for calculations which do not require great accuracy, we can use 3 to 4 groups). If lines are taken into consideration, the situation becomes more complicated. But here also we can use the same considerations and separate out the lines with the same contours [11]. Formally the method is applicable for arbitrary dependence of κ on ε , T and ρ . Furthermore, there is no doubt about its effectiveness in the majority of cases. In fact, "the additional work" of computing ψ_{ε}^{\pm} , c_{ε}^{\pm} , g_{ε}^{\pm} , $\varphi_{\varepsilon}^{\pm}$, $\langle \kappa \rangle^{\pm}$, c_i^{\pm} , g_i^{\pm} , η_i^{\pm} or ξ_i^{\pm} does not exceed greatly in volume the computation (by means of integrating J_{ε}) of quantities U_{ε}^{\pm} and U_i^{\pm} and it is less than the work of calculating J_{ε} . Therefore a saving in computational work arises even for quite frequent averaging through one computational layer. If, however, the averaging is carried out infrequently, then the simplification achieved is quite substantial and becomes even essential, because the solution of many nonsteady-state problems cannot be accomplished by direct integration of transfer equations, even with application of modern computational techniques. The effectiveness of the indicated method for many problems (for example, the interaction of powerful radiation fluxes with the surface of a solid body and the layer of vapor which is formed) is enhanced by the circumstance that from a gasdynamic point of view the processes which take place are slow: during the period of interaction the sonic perturbations have time for multiple propagation through the heated layer. This results in a relatively slowly changing temperature and density profile.

It should be noted that the described method can be generalized to the non-one-dimensional case and also to the case where local thermodynamic equilibrium is absent.

6. Now we shall examine the case where (5.1) is satisfied with sufficient accuracy in the entire range of ε , T and ρ , to be investigated.

If the radiation spectrum of the source is unchanged, it is sufficient to carry out the averaging of equations only once; if the spectrum changes slowly, it is sufficient to carry out the averaging infrequently and in the intervening moments of time it is possible to use interpolation for coefficients c and ξ . For an unchanging spectrum the problem of radiation action from a source of continuous spectrum will be reduced to the problem

of action from several (as far as the number of groups is concerned) equivalent lasers or one (in the case of integration over the entire spectrum) equivalent laser. These lasers will behave as if the energy of quanta changes in the process of propagation (in a manner known in advance).

Let the coefficient of absorption be a power function

$$K(\epsilon) = K(\epsilon/\epsilon_0)^s \tag{6.1}$$

Then we shall have for Planck's spectrum of the source

$$\langle \kappa \rangle^\circ = \kappa^p = \frac{15}{\pi^4} \kappa_T \Gamma(s-3) \zeta(s-3) \tag{6.2}$$

where $\Gamma(x)$ is the gamma function, $\zeta(x)$ is the Riemann zeta function and κ_T is the coefficient of absorption for $\epsilon = T$.

For the Rayleigh-Jeans spectrum (4.12) we shall have

$$\langle \kappa \rangle^\circ = 3\kappa_m / (3 - s) \tag{6.3}$$

The average energy of quanta of Planck's spectrum $\langle \epsilon \rangle^p = 4.037T$ and for (4.12), $\langle \epsilon \rangle^\circ = 3/4 \epsilon_m$. It is therefore apparent that in one and the other case there is noticeable deviation of the surface average coefficient of absorption from $\kappa(\langle \epsilon \rangle^\circ)$. This last quantity or coefficient of absorption, averaged over the optical thickness τ of the order of unity, corresponds better to the value of $\langle \kappa \rangle$ in that region where the principal part of the energy of the source is radiated.

In Fig. 5 the dependence of $\xi = \langle \kappa \rangle / \kappa(\langle \epsilon \rangle^\circ)$ on τ is shown for the case of integration over the entire spectrum. Initially the spectrum is a Planck's spectrum. The dashed line indicates the relationship for the case where the initial spectrum is "shelf-like"

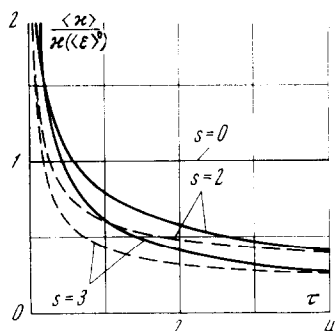


Fig. 5

$$\varphi_\epsilon = 1 / (\epsilon_m - \epsilon_0), \quad \epsilon_0 \leq \epsilon \leq \epsilon_m \tag{6.4}$$

The last case qualitatively demonstrates the character of the spectrum of a layer of multiply ionized plasma which was heated by laser radiation and which emits radiation from the volume. In this case $\langle \epsilon \rangle^\circ = 1/2(\epsilon_m + \epsilon_0)$.

The optical thickness is determined here in the following manner:

$$\tau = \int_0^m \kappa^\circ dm, \quad \kappa^\circ = \kappa(\langle \epsilon \rangle^\circ) \tag{6.5}$$

It is easy to present the solution of the averaged equation of transfer in the form which is analogous to the case of monochromatic directed radiation

$$q = q^\circ \exp(-\tau_e), \quad \tau_e = \int_0^{\bar{\tau}} \frac{\xi(\tau)}{c(\tau)} d\tau \tag{6.6}$$

Here τ_e is the effective optical thickness, q° is the radiation flux density of the source.

It follows from Fig. 5 that the relationship $\xi(\tau)$ does not show strong differences for substantially different spectra (Planck's spectra with a sharply defined maximum and shelf-like spectra without any maximum at all).

This gives rise to the hope that in other cases also (not only for the condition when (6.1) is satisfied) a smooth change in the shape of the spectrum from the source will not lead to a sharp change in the relationship $\xi(\tau)$ and that it will be necessary to

perform the averaging quite infrequently.

It also follows from Fig. 5 that for different (but qualitatively same) relationships $\kappa(\epsilon)$, namely for condition (6.1) in the case of $s = 2$ and $s = 3$, the relationship $\xi(\tau)$ does not change too strongly. Calculations according to (6.6) with utilization of relationships $\xi(\tau)$, represented in graphical form in Fig. 5, show that for $\tau \approx 0.5$ the flux density for the source of continuous spectrum (Planck's or shelf-like) for various s is approximately equal to the same quantity for a monochromatic source ($s = 0$). In addition $\xi \approx 1$ in this region, i. e. the intensity of energy release is also approximately equal

$$f^* = - \frac{\partial q}{\partial m} = \frac{\xi}{c} q^0 \kappa^0 \exp(-\tau_e) \tag{6.7}$$

We note that the principal part of the radiation energy is liberated in this very region.

For making rough initial estimates of magnitudes of parameters of a gas which is heated by a source of continuous spectrum the application of the "equivalency" principle can be recommended. This means that approximate and self-similar solutions of analogous problems on the action of a monochromatic radiation source (equivalent "laser")

are used. For example, methods and programs for numerical solution of such nonsteady-state problems are described in [6, 7, 12, 14]. In this connection analytical or tabulated relationships $\kappa(\epsilon_0, T, \rho)$, where ϵ_0 is the radiation energy of the monochromatic source, must be replaced by $\kappa(\langle \epsilon \rangle^0, T, \rho)$ or by $\kappa_{1/2}(T, \rho)$, which is the coefficient of absorption averaged over such an optical thickness that the radiation flux density $q = 1/2 q_0$.

In Fig. 6 the dependence of $\kappa_{1/2}$ (cm²/g) on temperature T (eV) and the relative density $\delta = \rho / \rho_L$ is shown for aluminum vapor. In this connection relationships $\kappa(\epsilon, T, \rho)$, which for some T and ρ were presented in Figs. 2-4, were averaged. The source spectrum was assumed to be a Planck spectrum at $\bar{T} = 8$ eV and "cut off" as a result of absorption ahead of the front of the radiating shock wave at $\epsilon = 24.6$ eV (a helium explosive source mentioned above).

Of course, when $\xi(\tau)$ is replaced by $\xi = 1$, details of distribution of the released energy over

the thickness of the heated material are represented inadequately, but many integral characteristics are determined correctly. In particular, this applies to the value of the average pressure in the layer of gas which is heated by radiation and which is expanding.

In paper [14] the author examined the self-similar problem of motion of a gas layer of constant mass which is heated continuously by radiation for the case where the intensity of energy release can be described by a function of the form

$$f = f^0(t)F(m) \tag{6.8}$$

Let κ be independent of ϵ and ρ , let the radiation spectrum of the source be invariable

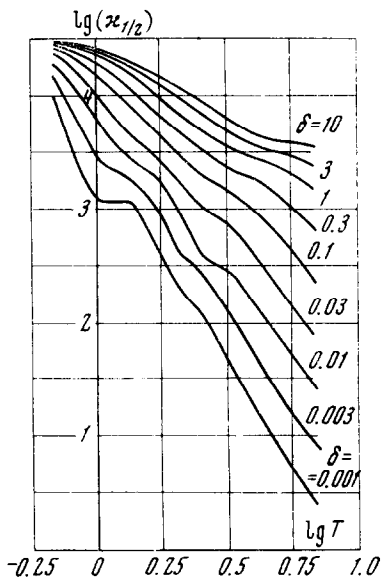


Fig. 6

with time. Then the characteristic mass of the heated layer of gas will not depend on its heating and motion and will remain constant. In this case equation (6. 8) is fulfilled, here

$$f^\circ = q^\circ \kappa^\circ, \quad F(\tau) = \frac{\xi(\tau)}{c(\tau)} \exp[-\tau_e(\tau)], \quad \tau = \kappa^\circ m \quad (6.9)$$

Let the radiation fall on the material from vacuum: $p = 0$ for $m = 0$. For the case $q^\circ = \text{const}$ it follows from the solution of the self-similar problem that the pressure on the fixed surface which forms the boundary of the heated gas in the point $m = m_0$ (total mass of the gas) is determined in the following manner:

$$p^\circ = \left(\frac{3q^\circ \kappa^\circ (\gamma - 1)}{2t(3\gamma - 1)} \right)^{1/2} P(M), \quad M = \kappa^\circ m_0 \quad (6.10)$$

The adiabatic exponent γ is assumed to be constant, M is the dimensionless mass of the heated layer. In this case the mass coincides with the optical thickness of the layer.

The value of the dimensionless pressure $P(M)$ is found from the solution of the problem

$$P \frac{d^2 P}{d\tau^2} = F(\tau); \quad P = 0, \quad \tau = 0$$

$$\frac{dP}{d\tau} = 0, \quad \tau = \kappa^\circ m_0 = M \quad (6.11)$$

In Fig. 7 results are presented for the solution of this equation for the case where $\xi(\tau)$ is a function shown in Fig. 5, i. e. for Planck's spectrum and a power dependence of $\kappa(\varepsilon)$. For simplicity we limit ourselves here to the case of $c = 1$. The results for a shelf-like spectrum (6. 4) are very close to the results of Planck's spectrum, and are therefore not shown in Fig. 7.

It is evident that values of $P(M)$ for $s = 2$ and $s = 3$ do not differ from each other strongly. Even values for the case $s = 0$ (monochromatic source or constant coefficient of absorption) obey a simple exponential law of energy release.

This result can be easily understood on the basis of the following.

The solution of (6. 11) can be presented in the form indicated in [14]

$$P(M) = \lambda \int_0^M \sqrt{F(\tau)} d\tau \quad (6.12)$$

If we set $\lambda = 1$, this will correspond to the absence of predistribution of energy released in the given particle of material in the process of hydrodynamic motion. For uniform energy release with respect to mass $\lambda = \lambda_0 = \sqrt{2/\pi} \approx 0.798$. For other cases λ can be found from solution of (6. 11). We point out that the difference between λ and λ_0 usually is not large (Fig. 7). It follows from (6. 12) that the value P (and p°) is determined by both the quantity of released energy and the mass of layers of material in which such energy release takes place. It follows from Fig. 5 that intensive energy release takes place at the surface ($\xi \gg 1$), but it occurs in a layer of small mass ($\tau \ll 1, m \ll 1/\kappa^\circ$). In deep layers of sufficiently great mass ($\tau \gg 1, m \gg 1/\kappa^\circ$) little energy is released. Therefore a significant part of the impulse is generated in

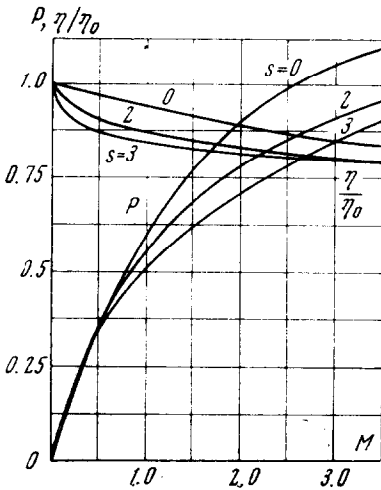


Fig. 7

layers $\tau \approx 1$, where $\langle \kappa \rangle \approx \kappa^0$ and $\xi \approx 1$.

In problems where it is required to determine the magnitude of pressure, it is necessary to take into account as accurately as possible those parts of the spectrum in which the quanta have sufficiently great penetrating capability, i. e. where the coefficient of absorption is small. The situation here is exactly opposite to problems on determination of radiation intensity emitted by a unit of volume, where the major role is usually played by those regions of spectra for which the coefficient of absorption is sufficiently high.

Analogous conclusions were reached by the author of [13] in the analysis of the self-similar problem of expansion of a gas heated by radiation with a continuous spectrum in the case where κ is a power function of e and ρ . This problem represents the generalization to the case of continuous spectrum, of problem [12] on propagation of a self-consistent wave of expansion and energy release through the medium which is heated by radiation from a monochromatic source. We note that in general for the case where (5.1) is satisfied and the spectrum remains invariant, all self-similar solutions on heating of the gas by monochromatic radiation have an analog (in any case formally) to the case of heating by a source with a continuous spectrum (in this connection, of course, the same additional requirements are made, for example, power form of $\kappa(e, \rho)$ and $\gamma = \text{const}$).

Realistically, the conditions of self-similarity usually are not satisfied completely (the adiabatic exponent γ is not constant, the absorption coefficient cannot be represented as a power function of e and ρ over the entire range of variation of these parameters etc.). However, the motion of the material frequently occurs in regimes which are close to self-similar. Consequently, it is required for such cases to perform the averaging quite infrequently, because in the self-similar problem the spectrum of radiation, the directional profile and with them the average cosines and the coefficients of distortion depend only on the self-similarity variable.

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DISCONTINUITY SURFACES IN DISPERSE SYSTEMS

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The equations of conservation of mass and momentum are considered for a fluid dispersing medium with suspended particles (the dispersible phase) along an arbitrary discontinuity surface of a disperse system. Conditions binding the velocity, pressure and concentration jumps are derived, and a model of surface tension at such surface is suggested. The coefficient of surface tension is dependent on the interrelation between the densities of phases, size of the dispersed phase particles, as well as on other parameters.

The problem of stability of the horizontal surface of a concentration discontinuity is solved. It is shown that, when a suspended layer is above the discontinuity surface, this surface is stable with respect to perturbations of sufficiently small wave length. The critical wave length, which defines the limit conditions of the onset of piston type fluidization, substantially depends on the effective surface tension. The upper free surface of the suspended layer remains, as expected, stable relative to perturbations of any wave length. The obtained results are in agreement with available experimental data.

A number of problems of mechanics of disperse systems reduce to the investigation of discontinuity surfaces. One of the most important among these is the determination of conditions for the occurrence of piston type fluidization which disrupts in the system the regular pattern of technological processes [1, 2]. The piston mode implies a sharp disruption of the suspended layer homogeneity, and can only be observed in sufficiently narrow tubes. It is characterized by a vertical stratification of a two-phase system into